Gears

Gear is a toothed member which is commonly used for transmitting motion by means of successively engaging teeth from a rotating shaft to another ~~a to~~ from a rotating shaft to a body which translates. In theory of gears, rolling contact will be there. So gear drive involves a higher member between driving and driven member.

The power which can be transmitted by rolling bodies is limited by the friction which can be developed at the surfaces. When the excessive load encountered, slippage occurs. Hence in order to provide a positive drive, teeth are placed on the contacting members and the resulting members are called gears.

Classification of Gears:-

Gears classification is done based on the relative orientation of the two shafts carrying gears.

(1) Gears Mounted on Parallel axes:-

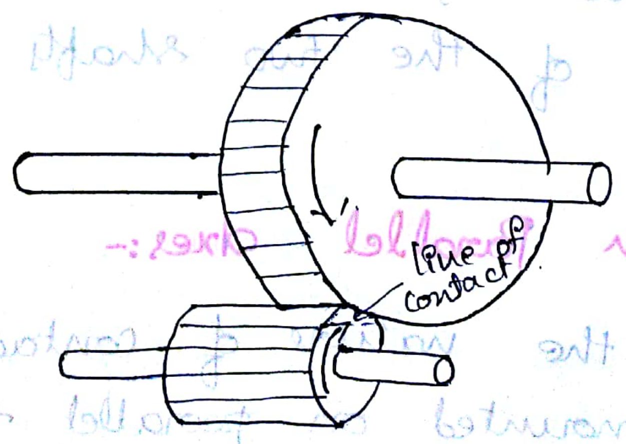
Irrespective of the nature of contact, a pair of gears mounted on parallel shaft produces a uniform motion that is equivalent to the rolling motion without slipping between two cylinders.

(i) Straight Spur Gears :- Gear pairs are having parallel axes of rotation, are spur gears. Spur gears have straight teeth parallel to the gear axes.

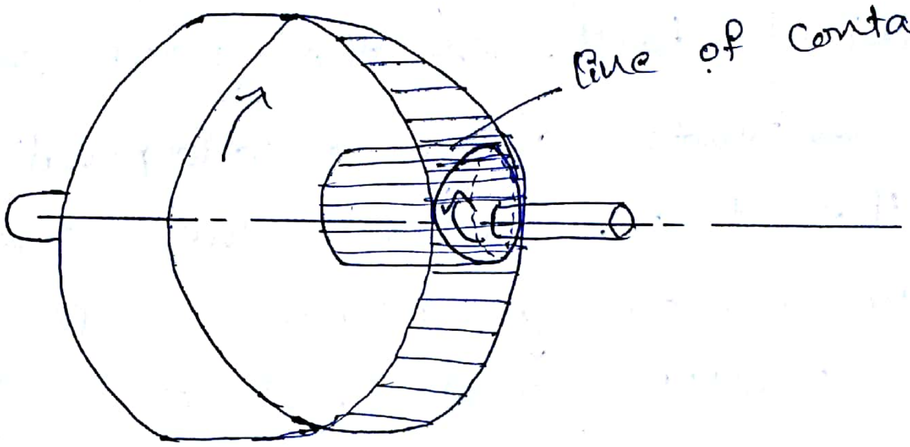
Spur gears have ~~parallel~~ straight teeth parallel to the axes and these are not subjected to thrust.

At the time of engagement of the two gears, the contact extends across the entire width on a line parallel to the axes of rotation. This results in sudden application of the load, high impact stresses and excessive noise at high speeds.

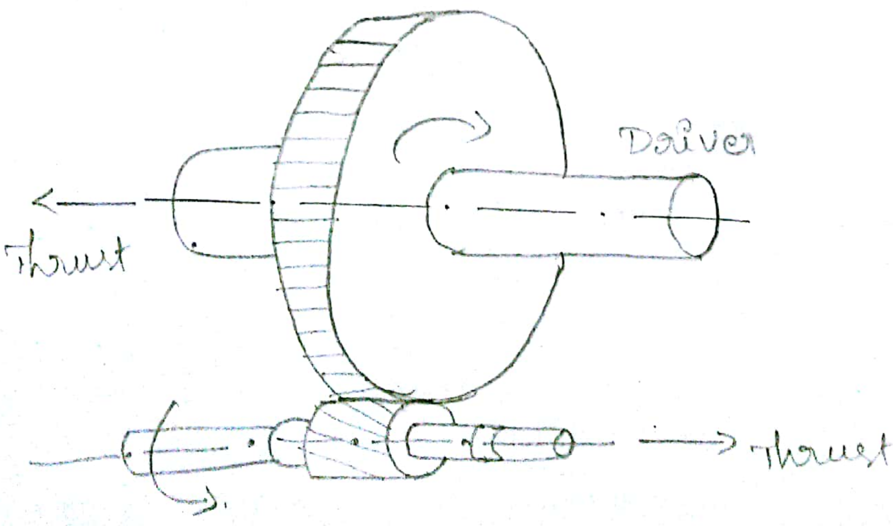
If the gears have external on the outer surface of the cylinders, the shafts rotate in the opposite direction.



In an internal spur gear, the teeth are formed on the inner surface of an annulus ring. An internal gear can mesh with an external pinion only and the two shafts rotate in the same direction.

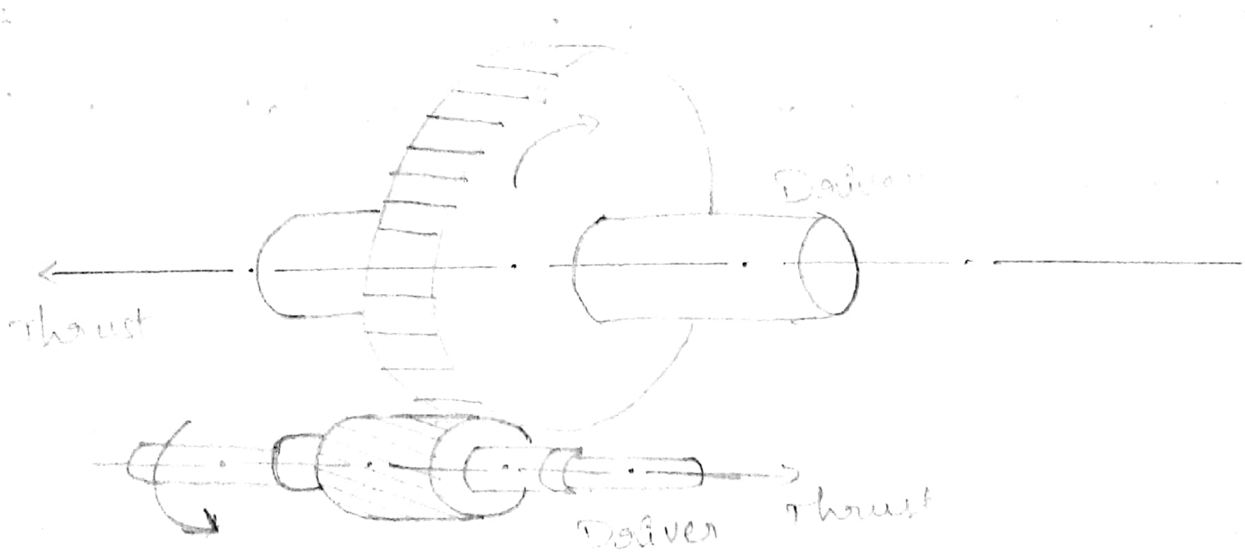


Spur rack and Pinion:- spur rack is a special case of spur gear where it is made of infinite diameter so that the pitch surface is a plane. Spur rack and pinion is used to convert rotary motion into translatory motion, or vice-versa.



Helical gears or Helical spur gears:- In helical gears, the teeth are curved and in helical shape. Two mating gears have the same helix angle, but have teeth of opposite hands.

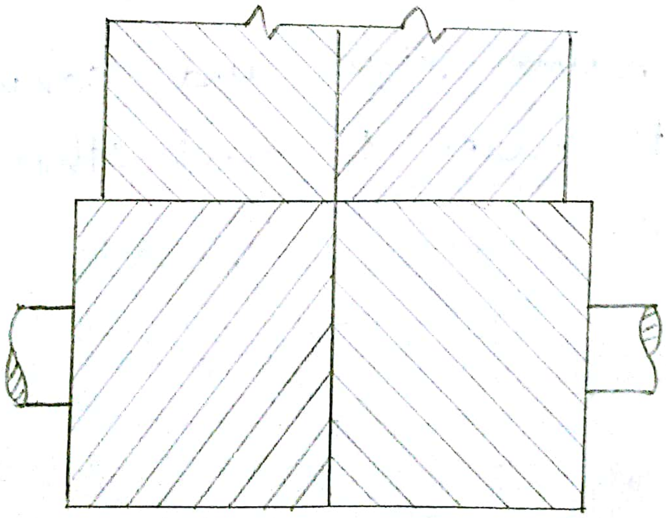
At the beginning of engagement, contact occurs only at the point of leading edge of curved teeth. As the gears rotate, the contact extends along a diagonal line across the teeth. Therefore the load is gradually increasing which results in low impact stresses and reduction in noise. Therefore, the helical gears can be used at higher velocities than the spur gears and have greater load carrying capacity.



Double Helical and Herringbone Gears:-

A double-helical gear is equivalent to a pair of helical gears secured together, one having a right-hand helix and the other a left-hand helix. The teeth of two rows are separated by a groove used for tool run out. Axial thrust of two rows of teeth cancel out each other. So, axial thrust can be eliminated. These can be run at high speeds with less noise and vibrations.

If the left and the right inclinations of a double-helical gear meet at common apex and there is no groove in between, the gear is known as herringbone gear.



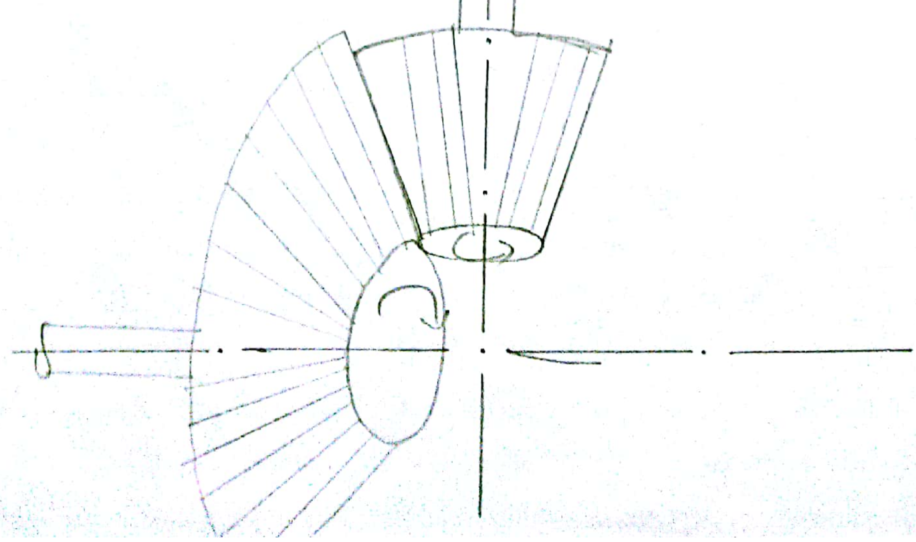
(2) Intersecting shafts:- The motion between two intersecting shafts is equivalent to the rolling of two cones, assuming no slipping. Generally, these gears are known as "Bevel gears".

When teeth are formed on the cones are straight, the gears are known as straight bevel and when inclined, they are known as spiral or helical bevel.

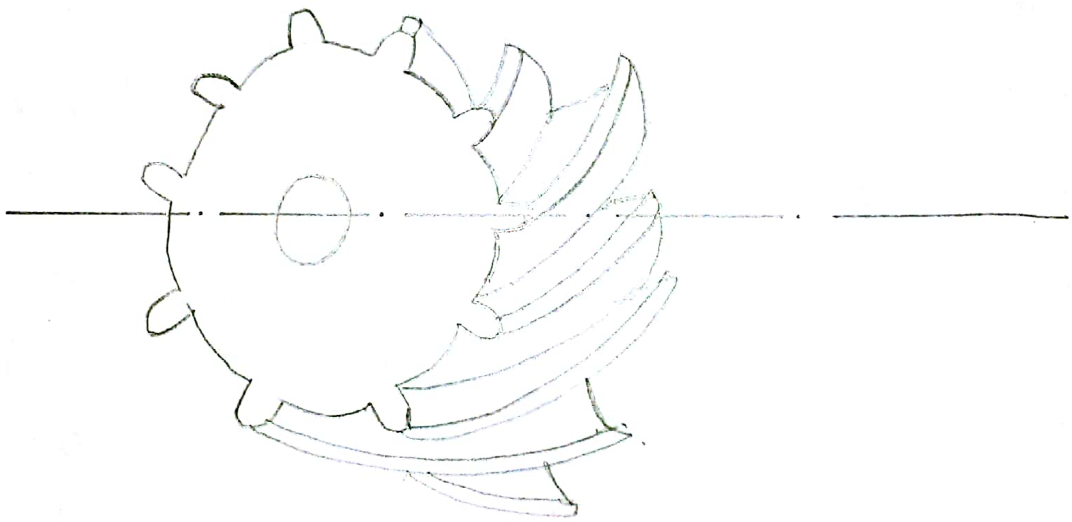
Straight Bevel Gears:-

The teeth are straight, radial to the point of intersection of the shaft axes and vary in cross section throughout their length. Generally bevel gears are used to connect shafts at right angles which run at low speeds.

* Gears of the same size and connecting two shafts at right angles to each other are known as mitre gears.

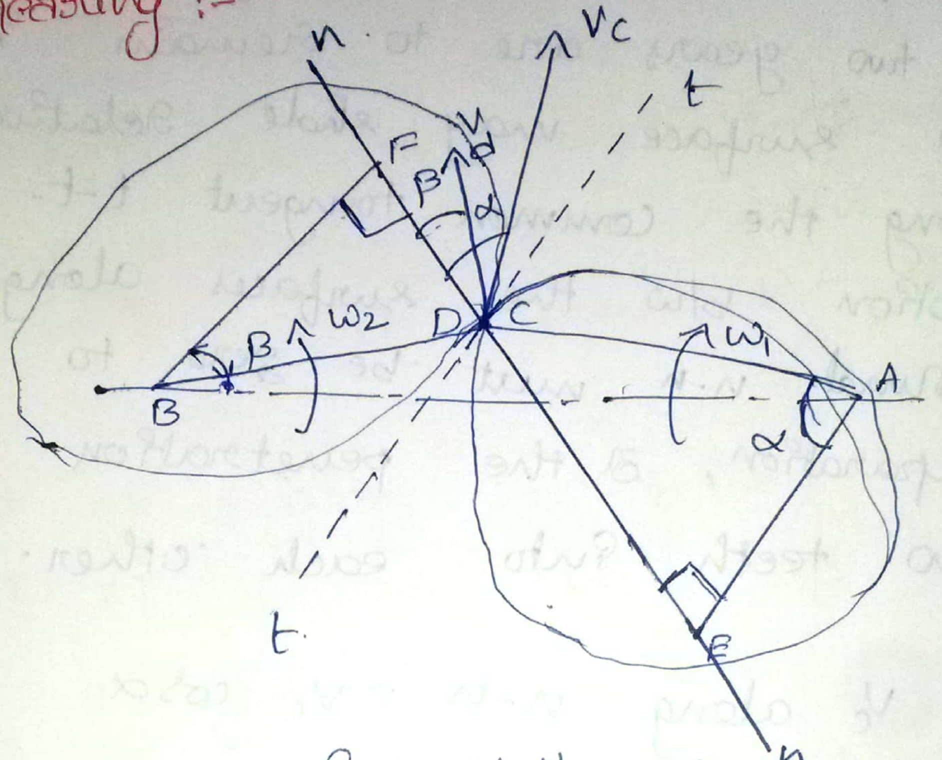


Spiral Bevel Gears:- When the teeth of a bevel gear are inclined at an angle to the face of the bevel, they are known as spiral bevels or helical bevels. They are smoother in action and quieter than straight tooth bevels as there is gradual load application and low impact stresses.



Zero Bevel Gears:- Spiral bevel gears with curved teeth but with a zero degree spiral angle are known as zero bevel gears.

Law of Gearing :-



The law of gearing states the condition which must be fulfilled by the gear tooth profile to maintain a constant angular velocity ratio between two gears.

A point C on the tooth profile of gear 1 is in contact with a point D on the tooth profile of the gear 2. Two curves in contact at points C & D must have a common normal (n-n) at the point.

ω_1 = instantaneous angular velocity of gear 1.

ω_2 = instantaneous angular velocity of gear 2.

V_c = linear velocity of C

V_d = linear velocity of D.

$V_c = \omega_1 \cdot AC$ $\perp^r$ to AC & inclined at α to n-n

$V_d = \omega_2 \cdot BD$ $\perp^r$ to BD & inclined at β to n-n

If the curved surfaces of the teeth of two gears are to remain in contact, one surface may slide relative to the other along the common tangent t-t. The relative motion b/w the surfaces along the common normal n-n must be zero to avoid the separation, or the penetration of the two teeth into each other.

V_c along n-n $= V_c \cos\alpha$.

V_d along n-n $= V_d \cos\beta$.

Relative motion along n-n $= V_c \cos\alpha - V_d \cos\beta$

Draw perpendiculars AE & BF on n-n.

Then $\angle CAE = \alpha$ and $\angle DBF = \beta$.

For proper contact,

$$V_c \cos\alpha - V_d \cos\beta = 0.$$

$$\omega_1 AC \cos\alpha - \omega_2 BD \cos\beta = 0$$

$$\omega_1 AC \frac{AE}{AC} - \omega_2 BD \frac{BF}{BD} = 0$$

$$\omega_1 AE - \omega_2 BF = 0.$$

$$\frac{\omega_1}{\omega_2} = \frac{\cancel{BB}\ BF}{AE} = \frac{BP}{AP} = \frac{FP}{EP}$$

$$\Rightarrow \frac{\omega_1}{\omega_2} = \frac{FP}{EP}$$

$$\Rightarrow \omega_1 EP = \omega_2 FP.$$

Velocity of sliding:-

If the curved surface of the two teeth of the gears 1 & 2 are to remain in contact, one can have a sliding motion relative to the other along the commont tangent t-t at C or D.

Component of V_c along t-$t = V_c \sin\alpha$.

Component of V_d along t-$t = V_d \sin\beta$.

velocity of sliding $= V_c \sin\alpha - V_d \sin\beta$.

$$= \omega_1 \cdot AC \cdot \frac{EC}{AC} - \omega_2 \cdot BD \cdot \frac{FD}{BD}$$

$$= \omega_1 EC - \omega_2 FD$$

$$= \omega_1 (EP + PC) - \omega_2 (FP - PD)$$

$$= \omega_1 EP + \omega_1 PC - \omega_2 FP + \omega_2 PC.$$

($\because$ C & D are coinciding points)

$$\cancel{= (\omega_1 + \omega_2) PC + (\omega_1 - \omega_2}$$

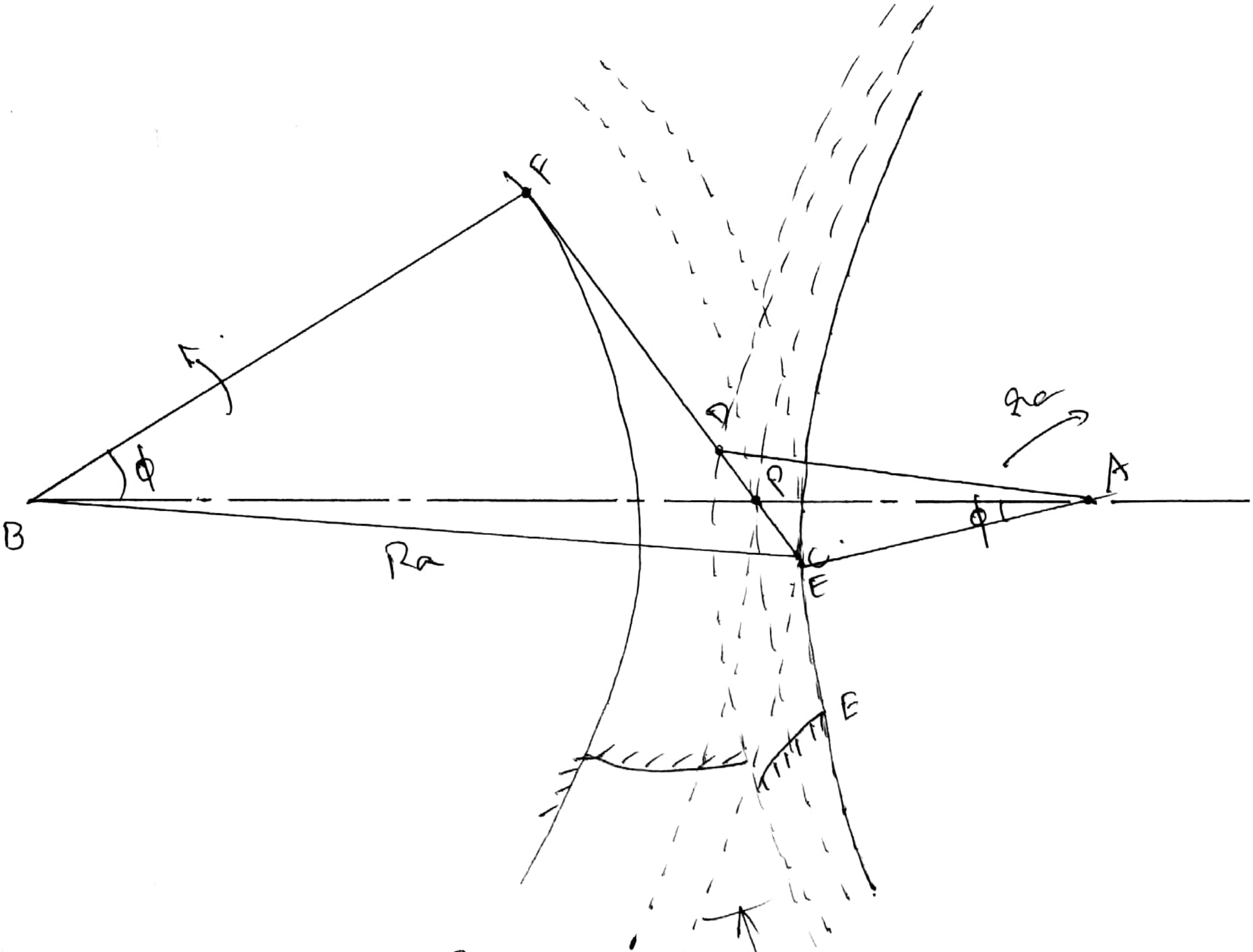
$$= (\omega_1 + \omega_2) PC + \omega_1 EP - \omega_2 FP.$$

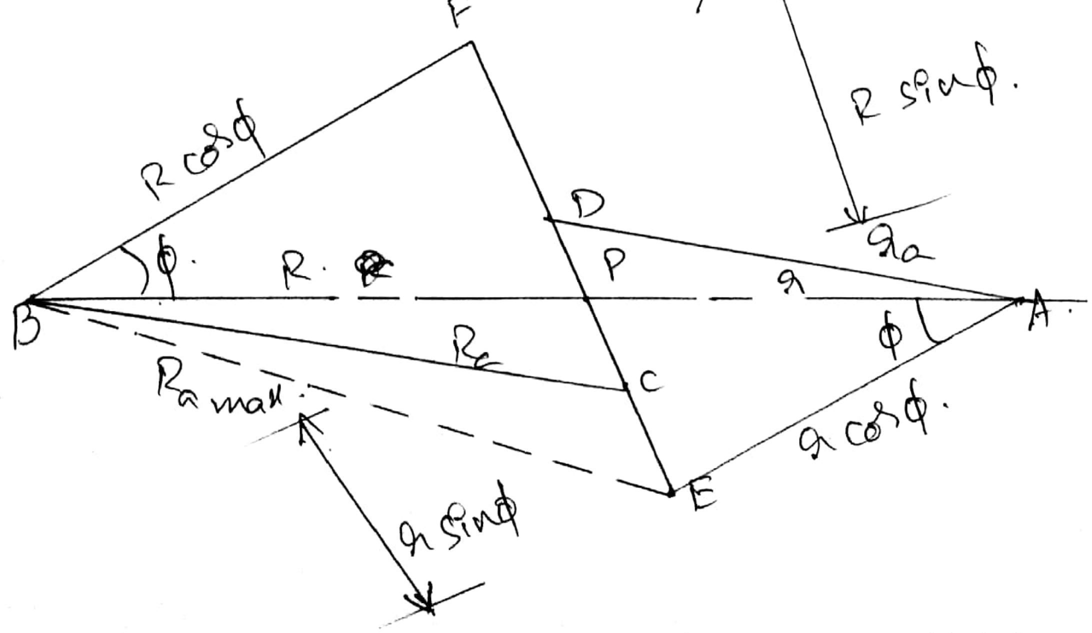
$$= (\omega_1 + \omega_2) PC.$$

velocity of sliding = sum of angular velocities × distance between the pitch point and the point of contact.

Path of Contact :-

Let the gear wheels with centres A & B be in contact.





CD ... The ... ...

The pinion 1 is the driver and rotating in clockwise direction. The wheel 2 is the driver and rotating in the counter clockwise direction. EF is their common tangent to the base circles.

Contact of the two teeth is made where the addendum circle of the wheel meets the line of action EF, i.e., at C and is broken where the addendum circle of the pinion meet the line of action, i.e., at D. CD is then the path of contact.

let

r = pitch circle radius of pinion.
R = pitch circle radius of wheel
r_a = addendum circle radius of pinion.
R_a = addendum circle radius of wheel.

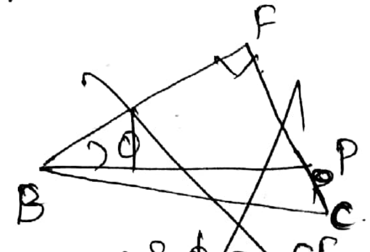
Path of contact = Path of Approach + Path of rece

$$CD = CP + PD$$

Path of Approach, CP = CF - PF.

~~$CP = BP\cos\phi - BP\sin\phi$~~

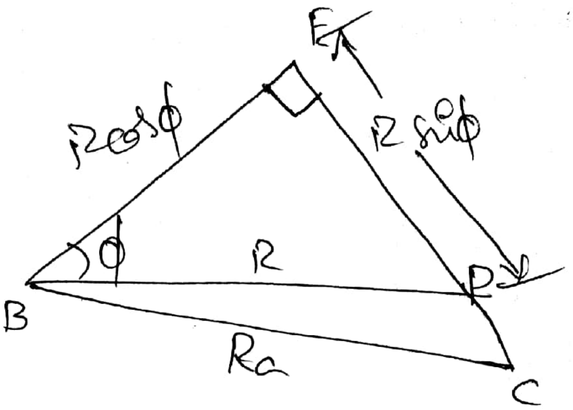
~~$= R\cos\theta$~~



$$\cos\phi = \frac{CF}{BP}$$

$$\sin\phi = \frac{PF}{BP}$$

Path of Approach, $CP = CF - PF$.



From Δ^{le} BFC,

$$BC^2 = BF^2 + CF^2$$

$$\Rightarrow CF = \sqrt{BC^2 - BF^2}$$

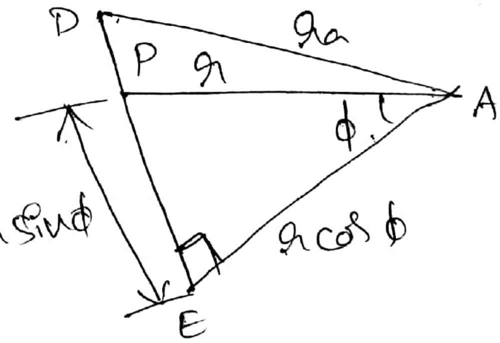
$$CF = \sqrt{R_a^2 - R^2\cos^2\phi}$$

$$PF = R\sin\phi.$$

$\therefore$ Path of Approach, $CP = CF - PF$

$$= \sqrt{R_a^2 - R^2\cos^2\phi} - R\sin\phi.$$

Path of recess, $PD = DE - PE$.



~~DE = DF~~

From Δ^{le} AED,

$$AD^2 = AE^2 + ED^2.$$

$$\Rightarrow DE = \sqrt{AD^2 - AE^2}$$

$$DE = \sqrt{r_a^2 - r^2\cos^2\phi}$$

$$PE = r\sin\phi.$$

$\therefore$ Path of recess, $PD = DE - PE$

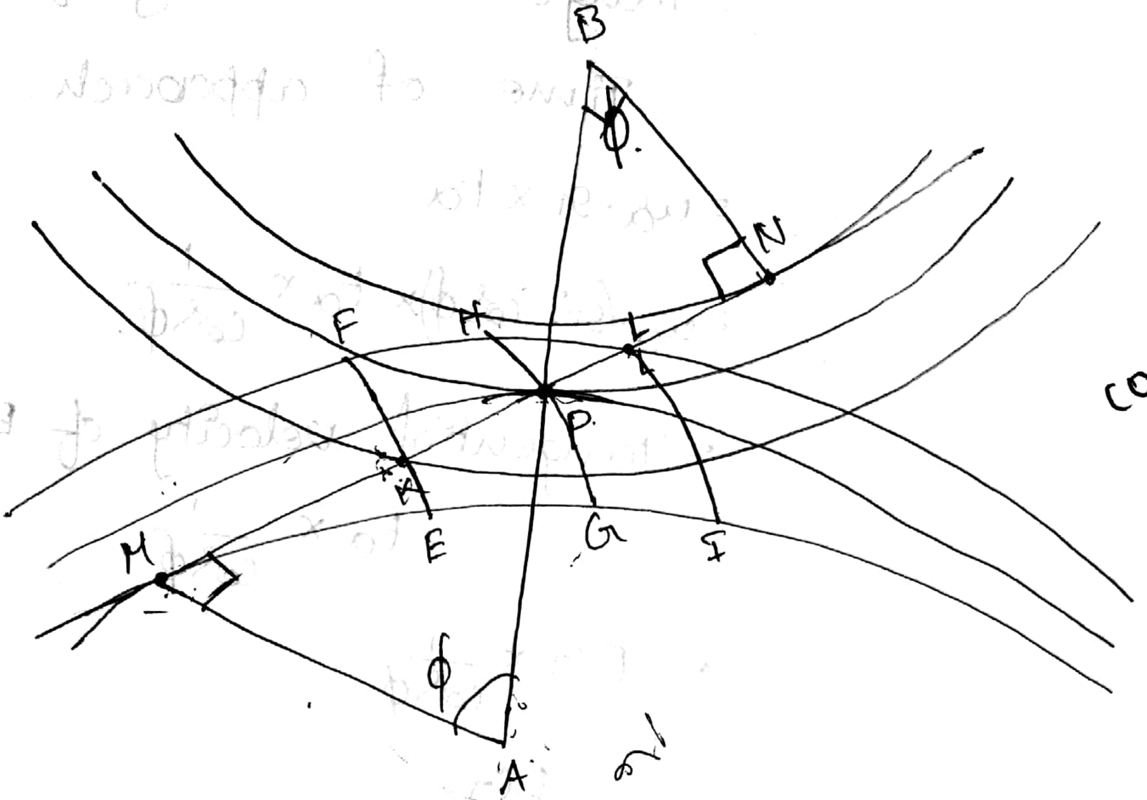
$$= \sqrt{r_a^2 - r^2\cos^2\phi} - r\sin\phi.$$

$\therefore$ Path of contact, $CD = CP + PD$.

$\therefore$ Path of contact $= \sqrt{R_a^2 - R^2\cos^2\phi} - R\sin\phi + \sqrt{r_a^2 - r^2\cos^2\phi} - r\sin\phi$

$$\boxed{\text{Path of contact} = \sqrt{R_a^2 - R^2\cos^2\phi} + \sqrt{r_a^2 - r^2\cos^2\phi} - (R+r)\sin\phi}$$

Arc of contact:-



$\cos\phi = \frac{MA}{r}$

$MA = r\cos\phi$

Arc of contact is the distance travelled by a point on either pitch circle of the two wheels during the period of contact of a pair of teeth.

At the beginning of engagement, the driving involute is EF, when the point of contact is P, it is GH, at the ending of engagement it is IL.

Arc of contact is P'P'' and it consists of arc of approach P'P and arc of recess PP''.

Arc of approach = $P'P$

$= \text{Tangential velocity of } P' \times \text{Time of approach.}$

$= \omega_a \cdot r \times t_a$

$= \omega_a (r \cos\phi) \times t_a \times \dfrac{1}{\cos\phi}$

$= \text{Tangential velocity of } E \times t_a \times \dfrac{1}{\cos\phi}$

$= EG \times \dfrac{1}{\cos\phi}$

$= \dfrac{EG}{\cos\phi}$

$= \dfrac{MG - ME}{\cos\phi}$

$= \dfrac{MP - MK}{\cos\phi}$

Arc of Approach $= P'P = \dfrac{PK}{\cos\phi}$

$\therefore$ Arc of Approach $= \dfrac{\text{Path of Approach}}{\cos\phi}$

Arc of Recess $= PP''$

$= \text{Tangential velocity } P \times \text{Time of Recess.}$

$= \omega_a \cdot r \times t_r$

$$PP'' = \omega_a (r\cos\phi) \frac{1}{\cos\phi} t_r$$

$$= (\text{Tangential velocity of } G)\, t_r \times \frac{1}{\cos\phi}$$

$$= GI \times \frac{1}{\cos\phi}$$

$$= \frac{\text{Arc } GI}{\cos\phi}$$

$$= \frac{\text{Arc } MI - \text{Arc } MG}{\cos\phi}$$

$$= \frac{ML - MP}{\cos\phi}$$

$$\text{Arc of Recess} = PP' = \frac{PL}{\cos\phi}$$

$$\text{Arc of Recess} = \frac{\text{Path of Recess}}{\cos\phi}$$

$$\therefore \text{Arc of contact} = \text{Arc of Approach} + \text{Arc of Recess}$$

$$= \frac{KP}{\cos\phi} + \frac{PL}{\cos\phi}$$

$$= \frac{KP + PL}{\cos\phi}$$

$$= \frac{KL}{\cos\phi}$$

$$\boxed{\text{Arc of Contact} = \frac{\text{Path of contact}}{\cos\phi}}$$

Number of Pairs of Teeth in contact (Contact Ratio):-

Number of teeth in contact, $n = \frac{\text{Arc of contact}}{\text{Circular Pitch}}$

$$n = \frac{KL}{\cos\phi} \cdot \frac{1}{P}$$

$\boxed{\text{Problem}}$:- Each of two gears in a mesh has 48 teeth and a module of 8 mm. The teeth are of 20° involute profile. The arc of contact is 2.25 times the circular pitch. Determine the addendum.

Sol:- $T_1 = T_2 = 48$, $m = 8\text{ mm}$, $\phi = 20°$

$$\text{Arc of contact} = 2.25 \times \text{circular pitch}$$

$$= 2.25 \times \frac{\pi d}{T}$$

$$= 2.25\pi m$$

$$= 2.25\pi \times 8 = 56.55\text{ mm}$$

$$\text{Path of contact} = \text{Arc of contact} \times \cos\phi$$

$$= 56.55 \times \cos 20° = 53.14\text{ mm}$$

$$\sqrt{R_a^2 - R^2\cos^2\phi} - R\sin\phi + \sqrt{r_a^2 - r^2\cos^2\phi} - r\sin\phi = 53.14$$

$$R = r = \frac{mT}{2} = \frac{8 \times 48}{2} = 192\text{ mm}$$

$$R_a = r_a$$

$$\Rightarrow 2\left[\sqrt{R_a^2 - R^2\cos^2\phi} - R\sin\phi\right] = 53.14$$

$$2\left[\sqrt{R_a^2 - 192^2 \cos^2 20^\circ} - 192 \sin 20^\circ\right] = 53.14$$

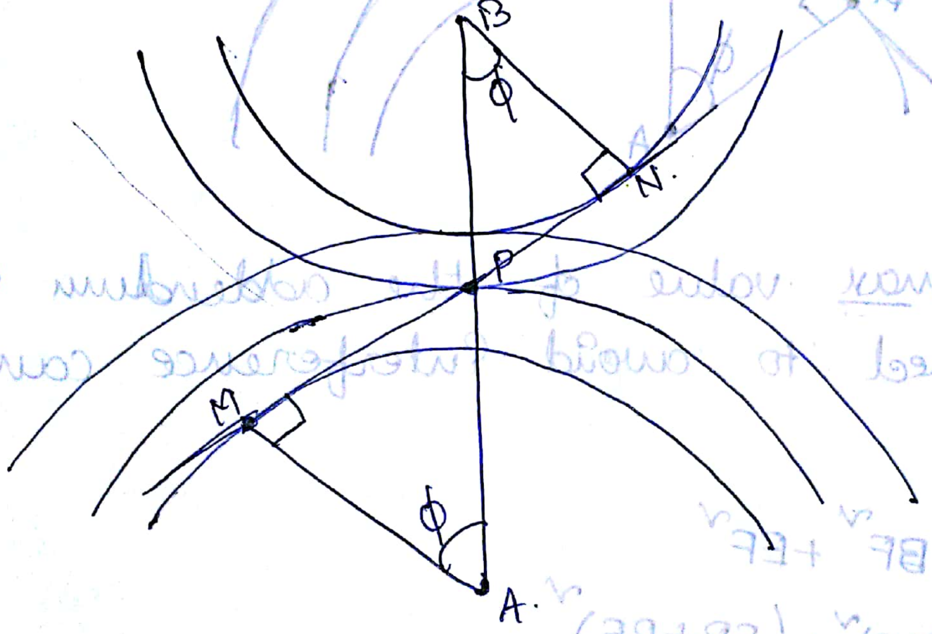
$$R_a = 202.6 \text{ mm}$$

$$\text{Addendum} = \cancel{R_a} = R_a - R$$

$$= 202.6 - 192 = 10.6 \text{ mm}$$

~~Minimum Number of Teeth:-~~

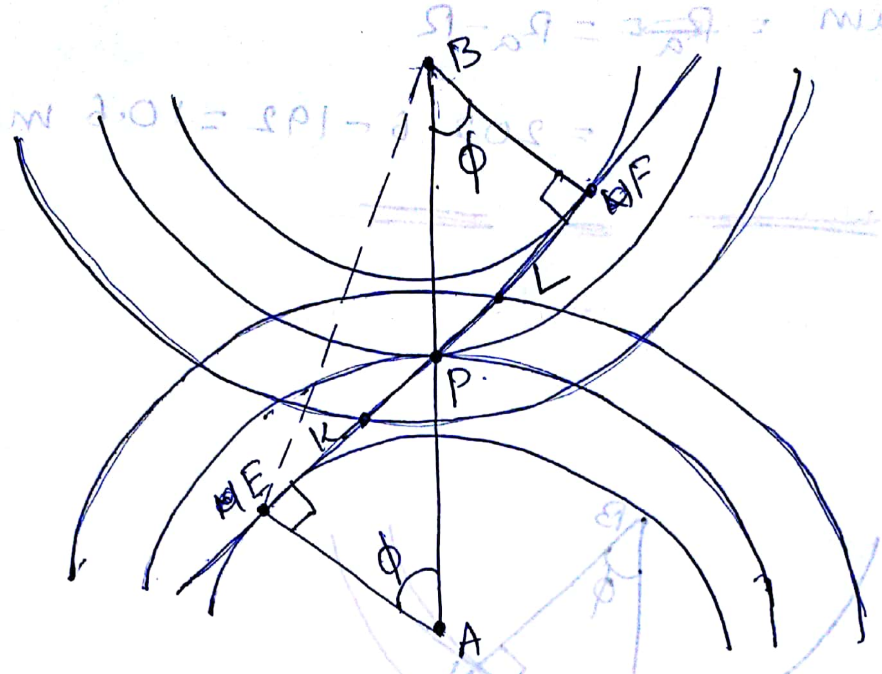
Interference in Involute Gears:-



Meshing of teeth means addendum of one teeth of one gear will insert into the deddendum of teeth of another gear.
We know that addendum will be equal to one module and deddendum will be equal to 1.15 times module. If addendum of wheel is more than deddendum of pinion then

interference will occur. M & N points are called interference points.

Minimum Number of Teeth:-



The max value of the addendum radius of the wheel to avoid interference can be upto BE.

$$BE^2 = BF^2 + EF^2$$

$$= (BF)^2 + (FP+PE)^2$$

$$= (R\cos\phi)^2 + (R\sin\phi + r\sin\phi)^2$$

$$= R^2\cos^2\phi + R^2\sin^2\phi + r^2\sin^2\phi + 2Rr\sin^2\phi$$

$$= R^2(\cos^2\phi + \sin^2\phi) + \sin^2\phi(r^2 + 2rR)$$

$$BE^2 = R^2 + (r^2 + 2rR)\sin^2\phi$$

$$BE^2 = R^2\left[1 + \frac{1}{R^2}(r^2 + 2rR)\sin^2\phi\right]$$

$$BE^2 = R^2\left[1+\left(\frac{r^2}{R^2}+\frac{2r}{R}\right)\sin^2\phi\right]$$

$$BE = R\sqrt{1+\frac{r}{R}\left(\frac{r}{R}+2\right)\sin^2\phi}$$

Maximum value of the addendum of the wheel can be equal to (BE - Pitch circle radius)

$$(a_w)_{max} = R\sqrt{1+\frac{r}{R}\left(\frac{r}{R}+2\right)\sin^2\phi} - R.$$

$$(a_w)_{max} = R\left[\sqrt{1+\frac{r}{R}\left(\frac{r}{R}+2\right)\sin^2\phi} - 1\right]$$

Let

t = number of teeth on the pinion

T = number of teeth on the wheel.

$R = \frac{mT}{2}$, $r = \frac{mt}{2}$, ~~Gear~~ Gear ratio, $G = \frac{T}{t}$

$$(a_w)_{max} = \frac{mT}{2}\left[\sqrt{1+\frac{mt/2}{mT/2}\left(\frac{mt/2}{mT/2}+2\right)\sin^2\phi} - 1\right]$$

$$= \frac{mT}{2}\left[\sqrt{1+\frac{t}{T}\left(\frac{t}{T}+2\right)\sin^2\phi} - 1\right]$$

$$(a_w)_{max} = \frac{mT}{2}\left[\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1\right]$$

$$\frac{mT}{2}\left[\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1\right] \geq a_w m.$$

$$T \geq \frac{2a_w}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi}-1}$$

$$T = \frac{2a_w}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi}-1}$$

This is minimum number of teeth on the wheel for the given values of the gear ratio, pressure angle and addendum coefficient a_w.

Minimum number of teeth on the pinion is

$$t = \frac{T}{G}$$

For $a_w = 1$, i.e addendum = module.

$$T \geq \frac{2}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi}-1}$$

if $t = T \Rightarrow G = 1$

$$T_{min} = \frac{2}{\sqrt{1+3\sin^2\phi}-1}$$

For $\phi = 20°$ $\Rightarrow$ $T_{min} = 12.31$

* In case of pinion, the maximum value of the addendum radius to avoid interference is AF.

$$(AF)^2 = (r\cos\phi)^2 + (R\sin\phi + r\sin\phi)^2$$

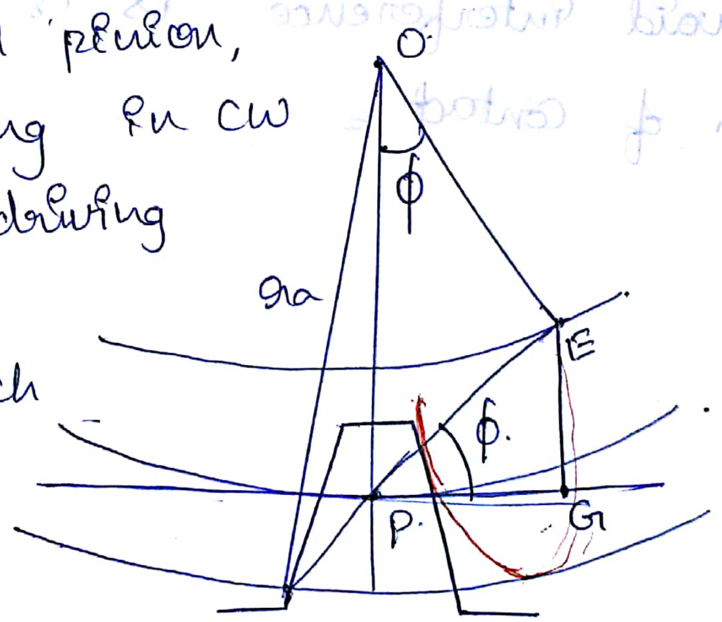
$$(a_P)_{max} = r\sqrt{1 + \frac{R}{r}\left(\frac{R}{r} + 2\right)\sin^2\phi} - r.$$

$$\boxed{(a_P)_{max} = \frac{mt}{2}\left[\sqrt{1 + G(G+2)\sin^2\phi} - 1\right].}$$

Interference between Rack and Pinion:-

In rack and pinion, pinion is rotating in CW direction and driving the rack.

P is the pitch point and PE is the line of action.



Addendum of the rack must be less than GE.

$$GE = PE\sin\phi = (r\sin\phi)\sin\phi = r\sin^2\phi.$$

$$GE = \frac{mt}{2}\sin^2\phi$$

To avoid interference,

$GE \geq a_r m.$

$\frac{mt}{2} \sin^2\phi \geq a_r m.$

$t \geq \frac{2a_r}{\sin^2\phi}.$

when $a_r = 1$

$\Rightarrow t_{min} \geq \frac{2}{\sin^2\phi}.$

For $\phi = 20^\circ$, $t_{min} = 17.1$ or 18.

$\therefore$ Minimum number of teeth on the pinion to avoid interference is 18.

Path of contact =

GEAR TRAINS

Gear Train is a combination of gears used to transmit motion from one shaft to another.

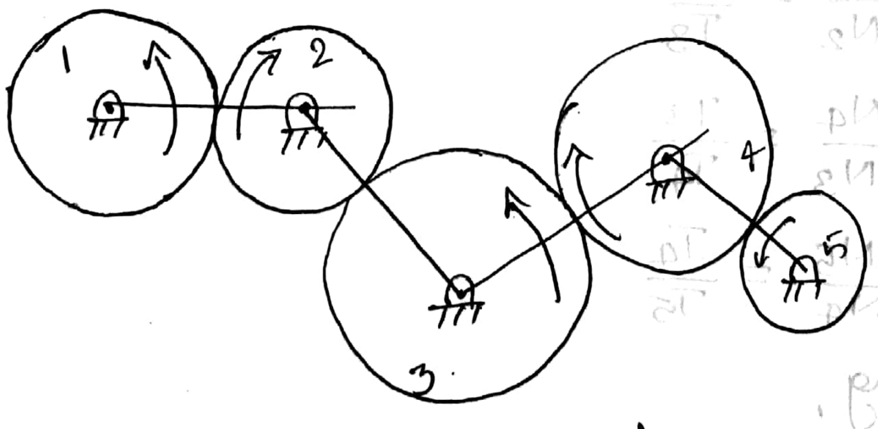
Main types of Gear Trains are:

1) Simple Gear Train
2) Compound Gear Train
3) Reverted Gear Train
4) Planetary or Epicyclic Gear Train

(1) Simple Gear Train:-

In simple gear trains, each shaft supports one gear.

Series of gears are arranged in such a way that gears can receive and transmit the motion from one gear to another gear.



→ Pair of external gears always move in opposite directions.

→ All odd-numbered gears move in one directio and all even-numbered gears move in opposite direction.

→ Speed ratio, is defined as the ratio of the speed of the driving shaft to that of the speed of the driven shaft.

→ Speed ratio is negative when the input and output gears rotate in opposite directions.

→ Speed ratio is positive when the input and output gears rotate in same direction

→ Train value :- Reciprocal of the speed ratio is known as Train value.

Let T = Number of teeth on a gear

N = speed of a gear in rpm.

$$\frac{N_2}{N_1} = \frac{T_1}{T_2}$$

$$\frac{N_3}{N_2} = \frac{T_2}{T_3}$$

$$\frac{N_4}{N_3} = \frac{T_3}{T_4}$$

$$\frac{N_5}{N_4} = \frac{T_4}{T_5}.$$

Multiplying,

$$\frac{N_2}{N_1} \times \frac{N_3}{N_2} \times \frac{N_4}{N_3} \times \frac{N_5}{N_4} = \frac{T_1}{T_2} \times \frac{T_2}{T_3} \times \frac{T_3}{T_4} \times \frac{T_4}{T_5}$$

$$\frac{N_5}{N_1} = \frac{T_1}{T_5}$$

Train value $\frac{N_5}{N_1} = \frac{T_1}{T_5}$

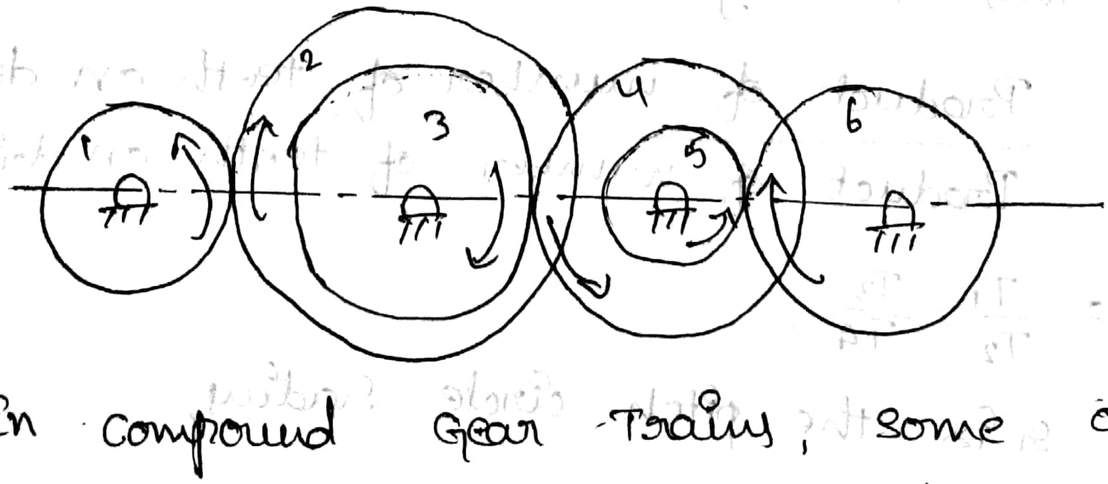
$= \frac{\text{Number of teeth on driving gear}}{\text{Number of teeth on driven gear.}}$

Speed ratio $= \frac{1}{\text{Train value.}}$

$$\frac{N_1}{N_5} = \frac{T_5}{T_1}$$

From this, we can say that intermediate gears have no effect on speed ratio. Therefore, they are known as "idlers."

Compound Gear Trains:-



In compound Gear Trains, some of the intermediate shafts carry more than one gea

If the gear 1 is the driver

$$\frac{N_2}{N_1} = \frac{T_1}{T_2}, \quad \frac{N_4}{N_3} = \frac{T_3}{T_4} \quad \text{and} \quad \frac{N_6}{N_5} = \frac{T_5}{T_6}$$

$$\frac{N_2}{N_1} \times \frac{N_4}{N_3} \times \frac{N_6}{N_5} = \frac{T_1}{T_2} \times \frac{T_3}{T_4} \times \frac{T_5}{T_6}$$

$$\frac{N_2}{N_1} \times \frac{N_4}{N_2} \times \frac{N_6}{N_4} = \frac{T_1}{T_2} \times \frac{T_3}{T_4} \times \frac{T_5}{T_6}$$

$$\frac{N_6}{N_1} = \frac{T_1}{T_2} \cdot \frac{T_3}{T_4} \cdot \frac{T_5}{T_6}$$

$$\text{Train value} = \frac{\text{Product of number of teeth on driving gears}}{\text{Product of number of teeth on driven gears.}}$$

Reverted Gear Train:-

If the axes of the first and last wheels of a compound gear coincide, it is called a reverted gear train.

This arrangement will be used in clocks.

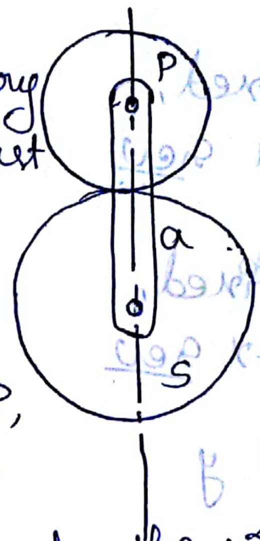
$$\frac{N_4}{N_1} = \frac{\text{Product of number of teeth on driving gears}}{\text{Product of number of teeth on driven gears}}$$

$$= \frac{T_1}{T_2} \cdot \frac{T_3}{T_4}.$$

if r is the pitch circle radius,

$$r_1 + r_2 = r_3 + r_4$$

Planetary or Epicyclic Gear Train :-



A gear train having a relative motion of axes is called a planetary or epicyclic gear train. Axis of atleast one of the gears also moves relative to the frame.

Consider two gear wheels S & P, the axes of which are connected by an arm. If arm a is fixed, the wheels S & P form a simple gear train.

If wheel S is fixed so that the arm can rotate about the axis of S, the wheel P also move around S. So it is epicyclic gear train.

Analysis :-

Assume that the arm a is fixed. Turn S through x revolutions in cw direction.

[cw dirn is +ve
ccw dirn is −ve]

revolutions made by $a = 0$

revolutions made by $S = x$.

revolutions made by $P = -\left(\frac{T_S}{T_P}\right)x$.

Let the locked system be turned through y revolutions in cw dirn.

Then

revolutions made by $a = y$

revolutions made by $S = x + y$

revolutions made by $P = y - \left(\frac{T_S}{T_P}\right)x$

Action	Revolutions of a	Revolutions of S	Revolutions of P.
a fixed, S +1 rev	0	1	$-\frac{T_s}{T_p}$
a fixed, S+x rev	0	x	$-\frac{T_s}{T_p}x$
Add y	y	y+x	$y-\frac{T_s}{T_p}x$

Relative Velocity Method :-

Angular velocity of S = Angular velocity of S relative to a + angular velocity of a.

$$\omega_s = \omega_{sa} + \omega_a.$$

$$N_s = N_{sa} + N_a$$

$$N_P = -N_{pa} + N_a$$ [∵ N_s & N_p are in opposite directions]

$$N_{sa} = N_s - N_a$$

$$N_{pa} = N_a - N_P$$

$$\frac{N_{sa}}{N_{pa}} = \frac{N_s - N_a}{N_a - N_p}.$$

$$\frac{T_p}{T_s} = -\frac{N_s - N_a}{N_p - N_a}$$

Unit -8

Gear trains

gear train :- some times, two or more gears are made to mesh with each other to transmit power from one shaft to another. Such a combination is called ‘ gear train’ or train of ‘ toothed wheels’.

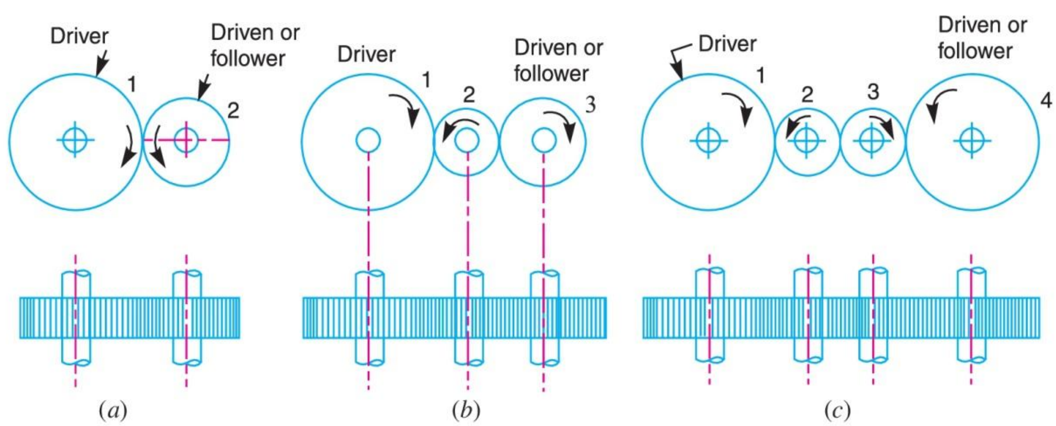
Types of gear trains :-

Following are the different types of gear trains, depending upon the arrangement of wheels:

1. simple gear train 2. Compound gear train 3. Reverted gear train and 4. Epicyclic gear train.

In the first three types of gear train, the axes of the shafts over which the gears are mounted are fixed relative to each other. But in case of epicyclic gear trains, the axes of the shafts on which the gears are mounted may move relative to a fixed axis.

1. simple gear train :-



Simple gear train.

When there is only one gear on each shaft, as shown in fig. it is known as simple gear train. The gears are represented by their pitch circles.

When the distance between the two shafts is small, the two gears 1 and 2 are made to mesh with each other to transmit motion from one shaft to the other, as shown in fig. sinec the gear 1 drives the gear 2, therefore gear 1 is called the ‘driver ‘ and the gear 2 is called the ‘ driven’ or ‘ follower’. It may be noted that the motion of the driven gear is opposite to the motion of driving gear.

Where

N_1 = speed of gear 1 (driver) in r.p.m

N_2= speed of gear 2 (driven or follower) in r.p.m

T_1= number of teeth on gear 1,and

T_2= number of teeth on gear 2.

Since the speed ratio (or velocity ratio) of gear train is the ratio of the speed of the driver to the speed of the driven and ratio of speeds of any pair of gears in mesh is the inverse of their number of teeth.

$$\text{Speed ratio} = \frac{N1}{N2} = \frac{T2}{T1}$$

It may be noted that ratio of the speed of the driven to the speed of the driver is known as train value of the gear train.

$$\text{Train value} = \frac{N2}{N1} = \frac{T1}{T2}$$

Some times, the distance between the two gears is large. In that case the following two methods are used.

1. by providing the large sized gears, or 2. By providing one or more intermediate gears.

It may be noted that when the number of intermediate gears are odd, the motions of both the gears is like as shown in fig.

But if the motion of intermediate gears are even, the motion of the driven will be in the opposite direction of the driver as shown in fig.

Now consider figure 2 :-

Let

N_1 = speed of driver in r.p.m

N_2= speed of intermediate gear in r.m.p

N_3 = speed of driven in r.p.m

T_1= number of teeth on driver

T_2= number of teeth on intermediate gear and

T_3= number of teeth on driven.

Since the driving gear 1 is in mesh with the intermediate gear 2, therefore speed ratio for these two gears is

$$\frac{N1}{N2} = \frac{T2}{T1} \ldots\ldots\ldots\ldots\ldots\ldots.1$$

Similary, intermediate gear 2 is in mesh with the driven gear 3, therefore speed ratio for these two gears is

$$\frac{N2}{N3} = \frac{T3}{T2} \ldots\ldots\ldots\ldots\ldots\ldots.2$$

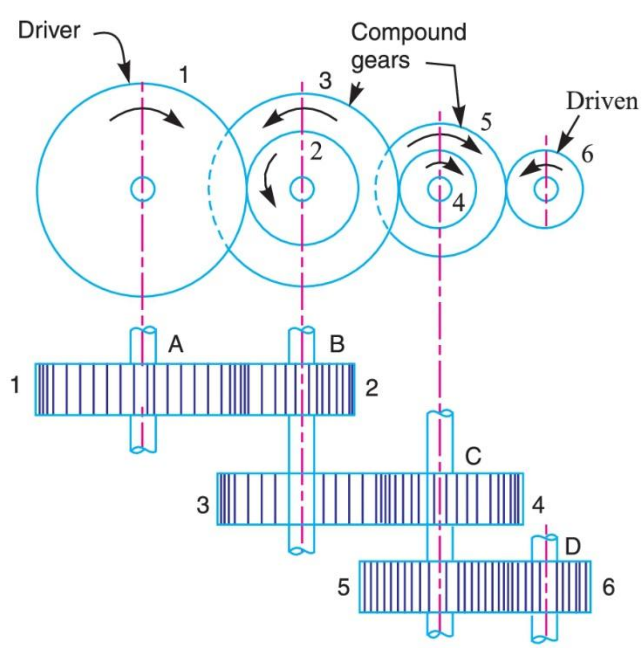
1 and 2 we get,

$$\therefore \frac{N1}{N3} = \frac{T3}{T1}$$

i.e., speed ratio = $\frac{speed\ \ of\ driver}{speed\ \ of\ driven} = \frac{number\ \ of\ teeth\ on\ driven}{number\ \ of\ teeth\ on\ driver}$

and train value = $\frac{speed\ \ of\ driven}{speed\ \ of\ driver} = \frac{number\ \ of\ teeth\ on\ driver}{number\ \ of\ teeth\ on\ driven}$

2. compound gear train :-



Compound gear train.

When there are more then one gear on a shaft, as shown in fig, it is called a compound train of gear. In a compound train of gears, as shown in figure. The gear 1 is the driving gear mounted on shaft A, gears 2 and 3 are compound gears which are mounted on shaft B. the gears 4 and 5 are also compound gears which are mound on shaft C and the gear 6 is the driven gear mounted on shaft D.

Let

N_1 = speed of driving gear 1

T_1= number of teeth on driving gear 1,

N_2, N_3 , …… N_6 =speed of respective gears in r.mp and

T2 , T3, ……. T6 = number of teeth on respective gears.

Since gear 1 is in mesh with gear 2 , therefore its speed ratio is $\frac{\mathrm{N1}}{\mathrm{N2}} = \frac{\mathrm{T2}}{\mathrm{T1}}$............1

Similary, for gears 3 and 4 speed ratio is $\frac{\mathrm{N3}}{\mathrm{N4}} = \frac{\mathrm{T4}}{\mathrm{T3}}$................................2

And for gears 5 and 6, speed ratio is $\frac{\mathrm{N5}}{\mathrm{N6}} = \frac{\mathrm{T6}}{\mathrm{T5}}$ 3

1,2 and 3 we get,

$$\frac{\mathrm{N1}}{\mathrm{N2}} \times \frac{\mathrm{N3}}{\mathrm{N4}} \times \frac{\mathrm{N5}}{\mathrm{N6}} = \frac{\mathrm{T2}}{\mathrm{T1}} \times \frac{\mathrm{T4}}{\mathrm{T3}} \times \frac{\mathrm{T6}}{\mathrm{T5}} \quad \text{OR} \quad \frac{\mathrm{N1}}{\mathrm{N6}} = \frac{\mathrm{T2}}{\mathrm{T1}} \times \frac{\mathrm{T4}}{\mathrm{T3}} \times \frac{\mathrm{T6}}{\mathrm{T5}} \quad (\mathrm{N2} = \mathrm{N3}, \mathrm{N4} = \mathrm{N5})$$

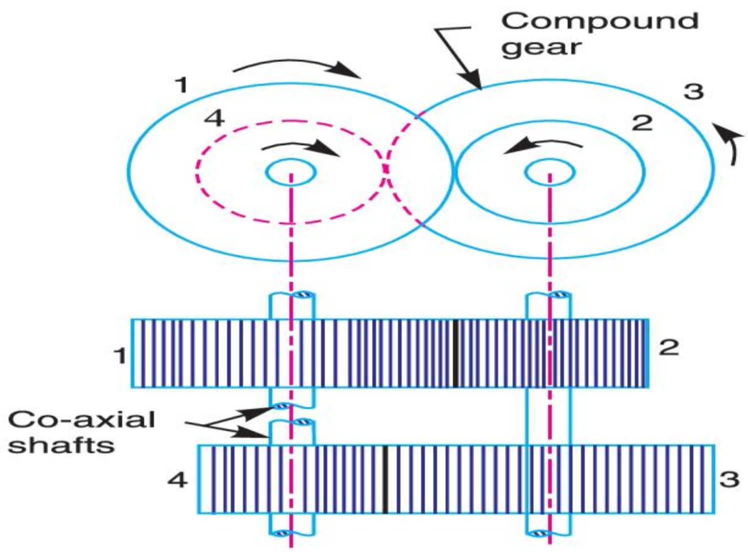
i.e., speed ratio = $\frac{speed\ \ of\ the\ frist\ driver}{speed\ \ of\ the\ last\ driven} = \frac{product\ \ of\ the\ number\ \ of\ teeth\ on\ drivens}{product\ \ of\ the\ number\ \ of\ teeth\ on\ drivers}$

and train value = $\frac{speed\ \ of\ the\ last\ driven}{speed\ \ of\ the\ first\ driver} = \frac{product\ \ of\ the\ number\ \ of\ teeth\ on\ drivers}{product\ \ of\ the\ number\ \ of\ teeth\ on\ drivens}$

3. reverted gear train :-

When the axes of the first gear (i.e., first driver) and the last gear (i.e., last driven) are co-axial, then the gear train is known as reverted gear trains shown in fig.

We see that gear 1 drivers the gear 2 in the opposite direction. Since the gears 2 and 3 are mounted on the same shaft, therefore they from a compound gear and the gear 3 will rotate in the same direction as that of gear 2. The gear 3 drives the gear 4 in the same direction as that of gear 1. Thus we see that in a reverted gear train, the motion of the first gear and the last gear is like.



Reverted gear train.

Let

T_1= number of teeth on gear 1,

r_1 = pitch circle radius of gear 1 and

N_1 = speed of gear 1 r.p.m

Similary,

T_2 , T_3, T_4 = number of teeth on respective gears.

r_2 , r_3, r_4 = pitch circle radii of respective geras, and

N_2, N_3 , N_4 = speed of respective gears in r.p.m

Since the distance between the centres of the shafts of gears 1 and 2 as well as gears 3 and 4 is same,

$$r1 + r2 = r3 + r4 \ldots\ldots\ldots\ldots\ldots\ldots.1$$

also, the circle pitch of all the gears is assumed to be same, therefore number of teeth on each gear is directly proportional to its circumference or radius.

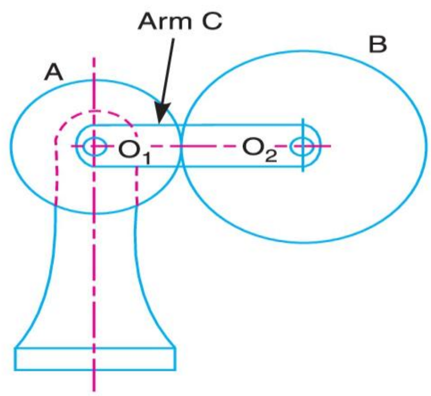
$$T1 + T2 = T3 + T4 \ldots\ldots\ldots\ldots\ldots..2$$

And speed ratio = $\frac{product \;\; of \; the \; number \;\; of \; teeth \; on \; drivens}{product \;\; of \; the \; number \;\; of \; teeth \; on \; drivers}$

$$\frac{N1}{N4} = \frac{T2}{T1} \times \frac{T4}{T3} \quad \ldots\ldots\ldots\ldots\ldots\ldots 3$$

The reverted gear trains are used in automobile trasmissions, lathe back gears, industrial speed reduces, and in clocks.

4. Epicyclic gear train :-



Epicyclic gear train.

,

Epicyclic gear train can be expressed, the axes of the shafts, over which the gears are mounted, may move relative to a fixed axis. A simple Epicyclic gear train is shown in figure. Where a gear A and the arm C have a common axis at 0_1, about which they can rotate. The gear B meshes with gear A and has its axis on the arm at 0_2, about which the gear B can rotate. If the arm is fixed, the gear train is simple and gear A can drive gear B or vice-versa, but if gear A is fixed and the arm is rotated about the axis of gear A (i.e., 0_1), then the gear B is forced to rotate upon and around gear A. such a motion is called Epicyclic and the gear trains arranged in such a mannner that one or more of their members move upon and around gear A. such a motion is called epicyclic and the gear trains arranged in such a manner that one or more of their members move upon and around another member are known as epicyclic gear trains (epi means upon and cyclic means around). The epicyclic gear trains may be simple or compound.

The epicyclic gear trains are useful for transmitting high velocity ratios with gears of moderate size in a comparatively lesser space. It is used automobiles, pulley blocks, wrist watches etc.

Velocity ratio of epicyclic gear train :-

The following two methods may be used for finding out the velocity ratio of an epicyclic gear train.

1. Tabular method. 2. Algebraic method

1.Tabular method:-

Consider an epicyclic gear train as shown in figure.

T_A= number of teeth on gear ‘A’, and T_B = number of teeth on gear ‘B’. first of all, let us suppose that the arm is fixed. Therefore the axis of both the gears are also fixed relative to each other. When the gear ‘A’ makes one relative to each other. When the gear ‘A’ makes one revolution anticlockwise, the gear B will T_A/T_B revolutions, clock wise. Assuming the anticlock wise rotation as positive and clock wise as negative, we may say that when gear A makes +1 revolution, then the gear B will make ($-T_A/T_B$) revolutions. This statement of relative motion is entered in the first row of the table.

Secondly, if the gear A makes +x revolutions, then the gear B will make –x (T_A/T_B) revolutions. This statement is entered in the second row of the table.

Thirdly, each element of an epicyclic train is given +y revolutions and entered in the third row. Finally, the motion of each element of the gear train is added up and entered in the fourth row.

Table of motions

Step No.	Conditions of motion	Revolutions of elements		
		Arm C	*Gear A*	*Gear B*
1.	Arm fixed-gear *A* rotates through + 1 revolution *i.e.* 1 rev. anticlockwise	0	$+1$	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear *A* rotates through + *x* revolutions	0	$+x$	$-x \times \frac{T_A}{T_B}$
3.	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_A}{T_B}$

2. algebraic method :-

In this method, the motion of each element of the epicyclic train relative to the arm is set down in the form of equations. The number of equations depends upon the number of elements in the gear train. But the two conditions are, usually, supplied in any epicyclic train viz, some elements is fixed and the other has specified motion. These two conditions are sufficient to slove all the equations, and hence to determine the motion of any element in the epicyclic gear train.

Let the cam C be fixed in an epicyclic gear train as shown in above epicyclic figure. Therefore speed of the gear A relative to the arm C.

$$= N_A - N_C$$

And speed of the gear B relative to the arm C,

$$= N_B - N_C$$

Since the gears A and B are meshing directly, therefore they will revolve in opposite directions.

$$\frac{N_B - N_C}{N_A - N_C} = \frac{-T_A}{T_B}$$

Since the arm C is fixed, therefore its speed , Nc= 0

$$\frac{N_B}{N_A} = \frac{-T_A}{T_B}$$

If the gear A is fixed, then $NA = 0$

$$\frac{N_B}{N_C} = 1 + \frac{T_A}{T_B}$$

Note:-

the gear at the center is called the sun gear and the gears whose axes moves are called planet gears.

Problems:-

Example 13.4. *In an epicyclic gear train, an arm carries two gears A and B having 36 and 45 teeth respectively. If the arm rotates at 150 r.p.m. in the anticlockwise direction about the centre of the gear A which is fixed, determine the speed of gear B. If the gear A instead of being fixed, makes 300 r.p.m. in the clockwise direction, what will be the speed of gear B ?*

Solution. Given : $T_A = 36$; $T_B = 45$; $N_C = 150$ r.p.m. (anticlockwise)

The gear train is shown in Fig. 13.7.

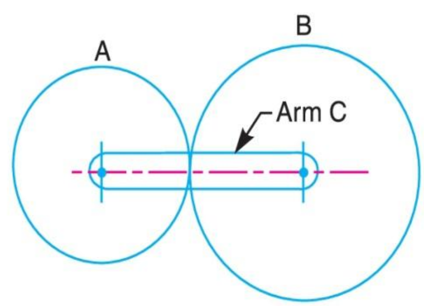


Fig. 13.7

We shall solve this example, first by tabular method and then by algebraic method.

1. ***Tabular method***

First of all prepare the table of motions as given below :

Table 13.2. Table of motions.

Step No.	Conditions of motion	Revolutions of elements		
		Arm C	Gear A	Gear B
1.	Arm fixed-gear A rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_A}{T_B}$
2.	Arm fixed-gear A rotates through + x revolutions	0	$+x$	$-x \times \frac{T_A}{T_B}$
3.	Add + y revolutions to all elements	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_A}{T_B}$

Speed of gear B when gear A is fixed

Since the speed of arm is 150 r.p.m. anticlockwise, therefore from the fourth row of the table,

$$y = +150 \text{ r.p.m.}$$

Also the gear A is fixed, therefore

$$x + y = 0 \quad \text{or} \quad x = -y = -150 \text{ r.p.m.}$$

$\therefore$ Speed of gear B, $N_B = y - x \times \frac{T_A}{T_B} = 150 + 150 \times \frac{36}{45} = +270$ r.p.m.

$= 270$ r.p.m. (anticlockwise) **Ans.**

Speed of gear B when gear A makes 300 r.p.m. clockwise

Since the gear A makes 300 r.p.m.clockwise, therefore from the fourth row of the table,

$$x + y = -300 \quad \text{or} \quad x = -300 - y = -300 - 150 = -450 \text{ r.p.m.}$$

$\therefore$ Speed of gear B,

$$N_B = y - x \times \frac{T_A}{T_B} = 150 + 450 \times \frac{36}{45} = +510 \text{ r.p.m.}$$

$= 510$ r.p.m. (anticlockwise) **Ans.**

2. ***Algebraic method***

Let N_A = Speed of gear A.

N_B = Speed of gear B, and

N_C = Speed of arm C.

Assuming the arm C to be fixed, speed of gear A relative to arm C

$$= N_A - N_C$$

and speed of gear B relative to arm $C = N_B - N_C$

Since the gears A and B revolve in ***opposite*** directions, therefore

$$\frac{N_B - N_C}{N_A - N_C} = -\frac{T_A}{T_B} \qquad ...(i)$$

Speed of gear B when gear A is fixed

When gear A is fixed, the arm rotates at 150 r.p.m. in the anticlockwise direction, *i.e.*

$$N_A = 0, \quad \text{and} \quad N_C = +150 \text{ r.p.m.}$$

$$\therefore \quad \frac{N_B - 150}{0 - 150} = -\frac{36}{45} = -0.8 \qquad \text{...[From equation } (i)]$$

or $$N_B = -150 \times -0.8 + 150 = 120 + 150 = 270 \text{ r.p.m.} \quad \textbf{Ans.}$$

Speed of gear B when gear A makes 300 r.p.m. clockwise

Since the gear A makes 300 r.p.m. clockwise, therefore

$$N_A = -300 \text{ r.p.m.}$$

$$\therefore \quad \frac{N_B - 150}{-300 - 150} = -\frac{36}{45} = -0.8$$

or $$N_B = -450 \times -0.8 + 150 = 360 + 150 = 510 \text{ r.p.m.} \quad \textbf{Ans.}$$

Example 13.5. *In a reverted epicyclic gear train, the arm A carries two gears B and C and a compound gear D - E. The gear B meshes with gear E and the gear C meshes with gear D. The number of teeth on gears B, C and D are 75, 30 and 90 respectively. Find the speed and direction of gear C when gear B is fixed and the arm A makes 100 r.p.m. clockwise.*

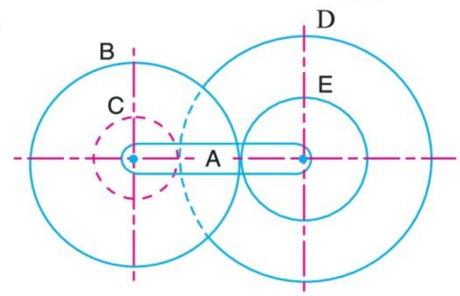


Fig. 13.8

Solution. Given : $T_B = 75$; $T_C = 30$; $T_D = 90$; $N_A = 100$ r.p.m. (clockwise)

The reverted epicyclic gear train is shown in Fig. 13.8. First of all, let us find the number of teeth on gear E (T_E). Let d_B, d_C, d_D and d_E be the pitch circle diameters of gears B, C, D and E respectively. From the geometry of the figure,

$$d_B + d_E = d_C + d_D$$

Since the number of teeth on each gear, for the same module, are proportional to their pitch circle diameters, therefore

$$T_B + T_E = T_C + T_D$$

$$\therefore \quad T_E = T_C + T_D - T_B = 30 + 90 - 75 = 45$$

The table of motions is drawn as follows :

Table 13.3. Table of motions.

Step No.	Conditions of motion	Revolutions of elements			
		Arm A	Compound gear D-E	Gear B	Gear C
1.	Arm fixed-compound gear *D-E* rotated through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_E}{T_B}$	$-\frac{T_D}{T_C}$
2.	Arm fixed-compound gear *D-E* rotated through + *x* revolutions	0	$+x$	$-x \times \frac{T_E}{T_B}$	$-x \times \frac{T_D}{T_C}$
3.	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_E}{T_B}$	$y - x \times \frac{T_D}{T_C}$

Since the gear *B* is fixed, therefore from the fourth row of the table,

$$y - x \times \frac{T_E}{T_B} = 0 \quad \text{or} \quad y - x \times \frac{45}{75} = 0$$

$$\therefore \qquad y - 0.6 = 0 \qquad \text{...}(i)$$

Also the arm *A* makes 100 r.p.m. clockwise, therefore

$$y = -100 \qquad \text{...}(ii)$$

Substituting $y = -100$ in equation **(*i*)**, we get

$$-100 - 0.6\,x = 0 \quad \text{or} \quad x = -100 / 0.6 = -166.67$$

From the fourth row of the table, speed of gear *C*,

$$N_C = y - x \times \frac{T_D}{T_C} = -100 + 166.67 \times \frac{90}{30} = +400 \text{ r.p.m.}$$

$$= 400 \text{ r.p.m. (anticlockwise)} \textbf{ Ans.}$$

Example 13.6. *An epicyclic gear consists of three gears A, B and C as shown in Fig. 13.10. The gear A has 72 internal teeth and gear C has 32 external teeth. The gear B meshes with both A and C and is carried on an arm EF which rotates about the centre of A at 18 r.p.m.. If the gear A is fixed, determine the speed of gears B and C.*

Solution. Given : $T_A = 72$; $T_C = 32$; Speed of arm EF = 18 r.p.m.

Considering the relative motion of rotation as shown in Table 13.5.

Table 13.5. Table of motions.

Step No.	*Conditions of motion*	*Revolutions of elements*			
		Arm EF	*Gear C*	*Gear B*	*Gear A*
1.	Arm fixed-gear C rotates through + 1 revolution (*i.e.* 1 rev. anticlockwise)	0	+ 1	$-\frac{T_C}{T_B}$	$-\frac{T_C}{T_B} \times \frac{T_B}{T_A} = -\frac{T_C}{T_A}$
2.	Arm fixed-gear C rotates through $+x$ revolutions	0	$+x$	$-x \times \frac{T_C}{T_B}$	$-x \times \frac{T_C}{T_A}$
3.	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y - x \times \frac{T_C}{T_B}$	$y - x \times \frac{T_C}{T_A}$

Speed of gear C

We know that the speed of the arm is 18 r.p.m. therefore,

$$y = 18 \text{ r.p.m.}$$

and the gear A is fixed, therefore

$$y - x \times \frac{T_C}{T_A} = 0 \quad \text{or} \quad 18 - x \times \frac{32}{72} = 0$$

$\therefore$ $x = 18 \times 72 / 32 = 40.5$

$\therefore$ Speed of gear C $= x + y = 40.5 + 18$

$= + 58.5$ r.p.m.

$= 58.5$ r.p.m. in the direction of arm. **Ans.**

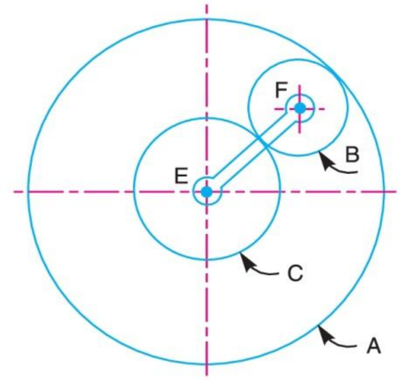


Fig. 13.10

Speed of gear B

Let d_A, d_B and d_C be the pitch circle diameters of gears A, B and C respectively. Therefore, from the geometry of Fig. 13.10,

$$d_B + \frac{d_C}{2} = \frac{d_A}{2} \quad \text{or} \quad 2\,d_B + d_C = d_A$$

Since the number of teeth are proportional to their pitch circle diameters, therefore

$$2\,T_B + T_C = T_A \quad \text{or} \quad 2\,T_B + 32 = 72 \quad \text{or} \quad T_B = 20$$

$\therefore$ Speed of gear B $= y - x \times \dfrac{T_C}{T_B} = 18 - 40.5 \times \dfrac{32}{20} = -46.8$ r.p.m.

$= 46.8$ r.p.m. in the opposite direction of arm. **Ans.**

Example 13.7. *An epicyclic train of gears is arranged as shown in Fig.13.11. How many revolutions does the arm, to which the pinions B and C are attached, make :*

1. *when A makes one revolution clockwise and D makes half a revolution anticlockwise, and*

2. *when A makes one revolution clockwise and D is stationary ?*

The number of teeth on the gears A and D are 40 and 90 respectively.

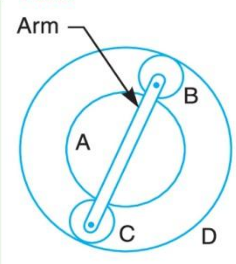


Fig. 13.11

Solution. Given : $T_A = 40$; $T_D = 90$

First of all, let us find the number of teeth on gears B and C (*i.e.* T_B and T_C). Let d_A, d_B, d_C and d_D be the pitch circle diameters of gears A, B, C and D respectively. Therefore from the geometry of the figure,

$$d_A + d_B + d_C = d_D \quad \text{or} \quad d_A + 2\,d_B = d_D \qquad ...(\because\ d_B = d_C)$$

Since the number of teeth are proportional to their pitch circle diameters, therefore,

$$T_A + 2\,T_B = T_D \quad \text{or} \quad 40 + 2\,T_B = 90$$

$$\therefore \quad T_B = 25, \quad \text{and} \quad T_C = 25 \qquad ...(\because\ T_B = T_C)$$

The table of motions is given below :

Table 13.6. Table of motions.

Step No.	Conditions of motion	Revolutions of elements			
		Arm	Gear A	Compound gear B-C	Gear D
1.	Arm fixed, gear A rotates through – 1 revolution (*i.e.* 1 rev. clockwise)	0	– 1	$+\frac{T_A}{T_B}$	$+\frac{T_A}{T_B}\times\frac{T_B}{T_D}=+\frac{T_A}{T_D}$
2.	Arm fixed, gear A rotates through $-x$ revolutions	0	$-x$	$+x\times\frac{T_A}{T_B}$	$+x\times\frac{T_A}{T_D}$
3.	Add $-y$ revolutions to all elements	$-y$	$-y$	$-y$	$-y$
4.	Total motion	$-y$	$-x-y$	$x\times\frac{T_A}{T_B}-y$	$x\times\frac{T_A}{T_D}-y$

1. ***Speed of arm when A makes 1 revolution clockwise and D makes half revolution anticlockwise***

Since the gear A makes 1 revolution clockwise, therefore from the fourth row of the table,

$$-x-y=-1 \quad \text{or} \quad x+y=1 \qquad \ldots(i)$$

Also, the gear D makes half revolution anticlockwise, therefore

$$x\times\frac{T_A}{T_D}-y=\frac{1}{2} \quad \text{or} \quad x\times\frac{40}{90}-y=\frac{1}{2}$$

$$\therefore \quad 40\,x-90\,y=45 \quad \text{or} \quad x-2.25\,y=1.125 \qquad \ldots(ii)$$

From equations *(i)* and *(ii)*, $x = 1.04$ and $y = -0.04$

$$\therefore \quad \text{Speed of arm} = -y = -(-0.04) = +0.04$$

$$= 0.04 \text{ revolution anticlockwise}$$ **Ans.**

2. ***Speed of arm when A makes 1 revolution clockwise and D is stationary***

Since the gear A makes 1 revolution clockwise, therefore from the fourth row of the table,

$$-x-y=-1 \quad \text{or} \quad x+y=1 \qquad \ldots(iii)$$

Also the gear D is stationary, therefore

$$x\times\frac{T_A}{T_D}-y=0 \quad \text{or} \quad x\times\frac{40}{90}-y=0$$

$$\therefore \quad 40\,x-90\,y=0 \quad \text{or} \quad x-2.25\,y=0 \qquad \ldots(iv)$$

From equations *(iii)* and *(iv)*,

$$x = 0.692 \quad \text{and} \quad y = 0.308$$

$\therefore$ Speed of arm $= -y = -0.308 = 0.308$ revolution clockwise **Ans.**

2. *Differential gear of an automobile*. The differential gear used in the rear drive of an automobile is shown in Fig. 13.21. Its function is

(*a*) to transmit motion from the engine shaft to the rear driving wheels, and

(*b*) to rotate the rear wheels at different speeds while the automobile is taking a turn.

As long as the automobile is running on a straight path, the rear wheels are driven directly by the engine and speed of both the wheels is same. But when the automobile is taking a turn, the outer wheel will run faster than the * inner wheel because at that time the outer rear wheel has to cover more distance than the inner rear wheel. This is achieved by epicyclic gear train with bevel gears as shown in Fig. 13.21.

The bevel gear A (known as pinion) is keyed to the propeller shaft driven from the engine shaft through universal coupling. This gear A drives the gear B (known as crown gear) which rotates freely on the axle P. Two equal gears C and D are mounted on two separate parts P and Q of the rear axles respectively. These gears, in turn, mesh with equal pinions E and F which can rotate freely on the spindle provided on the arm attached to gear B.

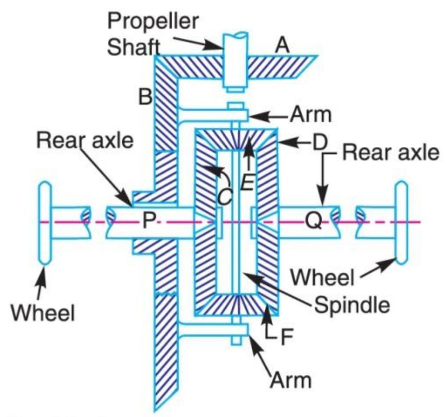


Fig. 13.21. Differential gear of an automobile.

When the automobile runs on a straight path, the gears C and D must rotate together. These gears are rotated through the spindle on the gear B. The gears E and F do not rotate on the spindle. But when the automobile is taking a turn, the inner rear wheel should have lesser speed than the outer rear wheel and due to relative speed of the inner and outer gears D and C, the gears E and F start rotating about the spindle axis and at the same time revolve about the axle axis.

Due to this epicyclic effect, the speed of the inner rear wheel decreases by a certain amount and the speed of the outer rear wheel increases, by the same amount. This may be well understood by drawing the table of motions as follows :

Table 13.17. Table of motions.

Step No.	*Conditions of motion*	*Revolutions of elements*			
		Gear B	*Gear C*	*Gear E*	*Gear D*
1.	Gear B fixed-Gear C rotated through + 1 revolution (*i.e.* 1 revolution anticlockwise)	0	+ 1	$+\frac{T_C}{T_E}$	$-\frac{T_C}{T_E}\times\frac{T_E}{T_D}=-1$ $(\because T_C=T_D)$
2.	Gear B fixed-Gear C rotated through $+x$ revolutions	0	$+x$	$+x\times\frac{T_C}{T_E}$	$-x$
3.	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y+x\times\frac{T_C}{T_E}$	$y-x$

From the table, we see that when the gear B, which derives motion from the engine shaft, rotates at y revolutions, then the speed of inner gear D (or the rear axle Q) is less than y by x revolutions and the speed of the outer gear C (or the rear axle P) is greater than y by x revolutions. In other words, the two parts of the rear axle and thus the two wheels rotate at two different speeds. We also see from the table that the speed of gear B is the mean of speeds of the gears C and D.

13.11. Torques in Epicyclic Gear Trains

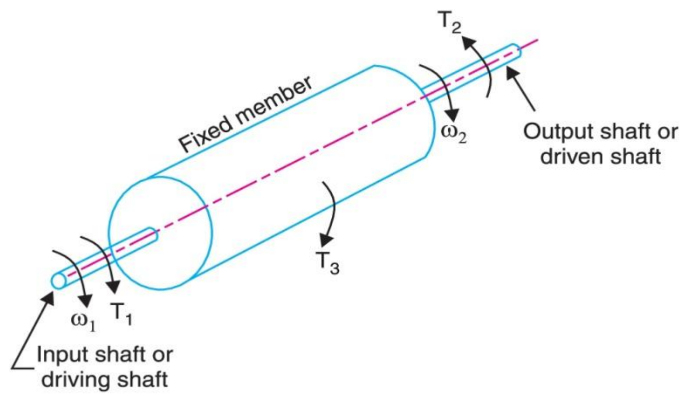


Fig. 13.25. Torques in epicyclic gear trains.

When the rotating parts of an epicyclic gear train, as shown in Fig. 13.25, have no angular acceleration, the gear train is kept in equilibrium by the three externally applied torques, *viz.*

1. Input torque on the driving member (T_1),

2. Output torque or resisting or load torque on the driven member (T_2),

3. Holding or braking or fixing torque on the fixed member (T_3).

The net torque applied to the gear train must be zero. In other words,

$$T_1 + T_2 + T_3 = 0 \quad \text{...}(i)$$

$$\therefore \quad F_1.r_1 + F_2.r_2 + F_3.r_3 = 0 \quad \text{...}(ii)$$

where F_1, F_2 and F_3 are the corresponding externally applied forces at radii r_1, r_2 and r_3.

Further, if ω_1, ω_2 and ω_3 are the angular speeds of the driving, driven and fixed members respectively, and the friction be neglected, then the net kinetic energy dissipated by the gear train must be zero, *i.e.*

$$T_1.\omega_1 + T_2.\omega_2 + T_3.\omega_3 = 0 \quad \text{...}(iii)$$

But, for a fixed member, $\omega_3 = 0$

$$\therefore \quad T_1.\omega_1 + T_2.\omega_2 = 0 \quad \text{...}(iv)$$

Notes : 1. From equations ***(i)*** and ***(iv)***, the holding or braking torque T_3 may be obtained as follows :

$$T_2 = -T_1 \times \frac{\omega_1}{\omega_2} \quad \text{...[From equation } (iv)]$$

and

$$T_3 = -(T_1 + T_2) \quad \text{...[From equation } (i)]$$

$$= T_1\left(\frac{\omega_1}{\omega_2} - 1\right) = T_1\left(\frac{N_1}{N_2} - 1\right)$$

2. When input shaft (or driving shaft) and output shaft (or driven shaft) rotate in the same direction, then the input and output torques will be in opposite directions. Similarly, when the input and output shafts rotate in opposite directions, then the input and output torques will be in the same direction.

Example 13.19. *Fig. 13.26 shows an epicyclic gear train. Pinion A has 15 teeth and is rigidly fixed to the motor shaft. The wheel B has 20 teeth and gears with A and also with the annular fixed wheel E. Pinion C has 15 teeth and is integral with B (B, C being a compound gear wheel). Gear C meshes with annular wheel D, which is keyed to the machine shaft. The arm rotates about the same shaft on which A is fixed and carries the compound wheel B, C. If the motor runs at 1000 r.p.m., find the speed of the machine shaft. Find the torque exerted on the machine shaft, if the motor develops a torque of 100 N-m.*

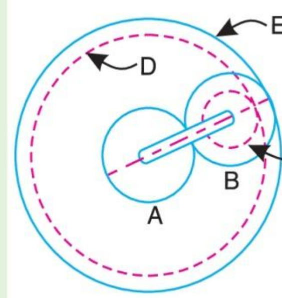


Fig. 13.26

Solution. Given : $T_A = 15$; $T_B = 20$; $T_C = 15$; $N_A = 1000$ r.p.m.; Torque developed by motor (or pinion A) = 100 N-m

First of all, let us find the number of teeth on wheels D and E. Let T_D and T_E be the number of teeth on wheels D and E respectively. Let d_A, d_B, d_C, d_D and d_E be the pitch circle diameters of wheels A, B, C, D and E respectively. From the geometry of the figure,

$$d_E = d_A + 2\, d_B \quad \text{and} \quad d_D = d_E - (d_B - d_C)$$

Since the number of teeth are proportional to their pitch circle diameters, therefore,

$$T_E = T_A + 2\, T_B = 15 + 2 \times 20 = 55$$

and

$$T_D = T_E - (T_B - T_C) = 55 - (20 - 15) = 50$$

Speed of the machine shaft

The table of motions is given below :

Table 13.21. Table of motions.

Step No.	Conditions of motion	Revolutions of elements				
		Arm	Pinion A	Compound wheel B-C	Wheel D	Wheel E
1.	Arm fixed-pinion A rotated through + 1 revolution (anticlockwise)	0	+ 1	$-\frac{T_A}{T_B}$	$-\frac{T_A}{T_B}\times\frac{T_C}{T_D}$	$-\frac{T_A}{T_B}\times\frac{T_B}{T_E}=-\frac{T_A}{T_E}$
2.	Arm fixed-pinion A rotated through + x revolutions	0	$+x$	$-x\times\frac{T_A}{T_B}$	$-x\times\frac{T_A}{T_B}\times\frac{T_C}{T_D}$	$-x\times\frac{T_A}{T_E}$
3.	Add + y revolutions to all elements	$+y$	$+y$	$+y$	$+y$	$+y$
4.	Total motion	$+y$	$x+y$	$y-x\times\frac{T_A}{T_B}$	$y-x\times\frac{T_A}{T_B}\times\frac{T_C}{T_D}$	$y-x\times\frac{T_A}{T_E}$

We know that the speed of the motor or the speed of the pinion A is 1000 r.p.m. Therefore

$$x + y = 1000 \quad \text{...(i)}$$

Also, the annular wheel E is fixed, therefore

$$y - x \times \frac{T_A}{T_E} = 0 \quad \text{or} \quad y = x \times \frac{T_A}{T_E} = x \times \frac{15}{55} = 0.273\,x \quad \text{...(ii)}$$

From equations (*i*) and (*ii*),

$$x = 786 \quad \text{and} \quad y = 214$$

∴ Speed of machine shaft = Speed of wheel D,

$$N_D = y - x \times \frac{T_A}{T_B} \times \frac{T_C}{T_D} = 214 - 786 \times \frac{15}{20} \times \frac{15}{50} = +37.15 \text{ r.p.m.}$$

$$= 37.15 \text{ r.p.m. (anticlockwise)}$$ **Ans.**

Torque exerted on the machine shaft

We know that

Torque developed by motor × Angular speed of motor

= Torque exerted on machine shaft × Angular speed of machine shaft

or $100 \times \omega_A$ = Torque exerted on machine shaft × ω_D

∴ Torque exerted on machine shaft

$$= 100 \times \frac{\omega_A}{\omega_D} = 100 \times \frac{N_A}{N_D} = 100 \times \frac{1000}{37.15} = 2692 \text{ N-m}$$ **Ans.**